**Monte Carlo Simulation**

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**INTRODUCTION**

Linear algebra is simply a method for solving systems of equations efficiently, and also appreciate that advanced linear algebra methods are core to machine learning. Let’s dive into our topic **Monte Carlo Simulation.** Monte Carlo simulations are used to model the probability of different outcomes in a process that cannot easily be predicted due to the intervention of random variables. It is a technique used to understand the impact of risk and uncertainty in prediction and forecasting models. Monte Carlo simulation can be used to tackle a range of problems in virtually every field such as finance, engineering, supply chain, and science. Monte Carlo simulation is also referred to as multiple probability simulation.

To achieve the simulation by Monte Carlo method, we use Cholesky decomposition to process the matrices. A Cholesky matrix transforms a vector of uncorrelated (i.e. independent) normally-distributed random variates into a vector of correlated (i.e. dependent) normally-distributed random variates. These now correlated random variates can be used in a Monte Carlo simulation where correlated random variates are required.

The last concept we use to solve problems of Monte Carlo Simulation is Positive Definite Matrix. Positive definite matrix is a symmetric matrix with all positive eigenvalues and as we are talking about symmetric matrices the all the eigenvalues real values.  
Monte Carlo methods are especially useful for simulating phenomena with significant uncertainty in inputs and systems with many coupled degrees of freedom. Areas of application include: Physical Sciences, Climate change and radiative forcing, Artificial intelligence for games, Finance and business and in the field of Mathematics its applications include Integration, Simulation and optimization, Inverse problems.

**REVIEW OF LITERATURE**

Paper 1:

**Introduction to Monte Carlo Simulation**

*Authors:**Samik Raychaudhuri*

*Proceedings of the 2008 Winter Simulation Conference S. J. Mason, R. R. Hill, L. Mönch, O. Rose, T. Jefferson, J. W. Fowler eds.*  
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*Abstract:*  
*The paper deals with providing an overview on Monte Carlo simulation and sheds light on* its applications. Monte Carlo simulation is a useful mathematical technique for analysing uncertain scenarios and providing probabilistic analysis of different situations. The basic principle for applying Monte Carlo analysis is simple and easy to grasp.   
It is a type of simulation that relies on repeated random sampling and statistical analysis to compute the results. This method of simulation is very closely related to random experiments, experiments for which the speciﬁc result is not known in advance.  
Models typically used depend on a number of input parameters, which when processed through the mathematical formulas in the model, results in one or more outputs. In most circumstances, experimenters develop several versions of the model, which can include the base case, the best possible scenario, and the worst possible scenario for the values of the input variables.  
This approach has various disadvantages. First, it might be difﬁcult to evaluate the best and worst case scenarios for each input variable. Second, all the input variables may not be at their best or worst levels at the same time. Decision making tends to be difﬁcult as well, since now we are considering more than one scenario. An experimenter might be tempted to run various ad-hoc values of the input parameters, often called what-if analysis, but it is not practical to go through all possible values of each input parameter. Monte Carlo simulation can help an experimenter to methodically investigate the complete range of risk associated with each risky input variable.  
There are four steps which are taken into consideration when performing Monte Carlo method. The first step is always developing a deterministic model which closely resembles the real scenario. In this deterministic model, we use the most likely value (or the base case) of the input parameters. We apply mathematical relationships which use the values of the input variables, and transform them into the desired output.  
When we are satisﬁed with the deterministic model, we add the risk components to the model and refer to the historical data of each input variable.  
After we have identiﬁed the underlying distributions for the input variables, we generate a set of random numbers from these distributions. One set of random numbers, consisting of one value for each of the input variables, will be used in the deterministic model, to provide one set of output values. We then repeat this process and collect different sets of possible output values. This part is the core of Monte Carlo simulation.  
After we have collected a sample of output values in from the simulation, we perform statistical analysis on those values.

Paper 2:

**New formulation of Cholesky decomposition and applications in stochastic simulation**

*Authors: Guoqing Huang, Haili Liao,* [*Mingshui Li*](https://www.sciencedirect.com/science/article/abs/pii/S0266892013000416#!)

*Published In: Probabilistic Engineering Mathematics (Volume 34, October 2013, Pages 40-47).*

*Abstract:*

This paper deals with the detection of wind characteristics when constructing bridges in different environments like rivers, ravines and other types of topographies. In particularly, it provides an improved method for predicting wind characteristics in different areas using Monte Carlo Simulation. It talks about the previously proposed methods for processing matrices for simulation and prediction. A novel formulation of the Cholesky decomposition for the EPSD/PSD matrix and associated simulation scheme is proposed. The central idea of this framework was that the complex EPSD/PSD matrix could be transformed into the real modulus matrix after the phase of wind was separated. This transformation required that the phase meet a pre-condition, which was generally satisfied for ground motion and wind field. With this treatment, the efficiency of decomposition was enhanced by four times compared with the direct Cholesky decomposition of the complex EPSD/PSD matrix. Also, the storage memory could be saved by about 50%. In the proposed simulation scheme, the separated phase could be easily assembled.  
The modulus of EPSD/PSD matrix could be further converted to the lagged coherence matrix, which depicted the coherence structure of stochastic process, shed more physical meaning and also facilitated stochastic simulation. In Cholesky decomposition based spectral representation method, the lagged coherence matrix was only decomposed on frequency domain and significantly improved the efficiency due to its time-independent characteristics, while the time-dependent EPSD matrix and modulus of EPSD matrix should be decomposed on both of time and frequency domains.  
In this paper, the application of the new method to Gaussian stochastic simulation was also presented. Firstly, the proposed approach facilitated extending the previous closed-form algorithm proposed by Yang et al. for wind field simulation into a more general situation, where auto EPSD/PSD could be different and also the wave passage effect could be included. Secondly, the proposed approach facilitated the application of interpolation techniques.

**REPORT ON THE PRESENT INVESTIGATION**

There is no consensus on how Monte Carlo should be defined. For example, Ripley defines most probabilistic modeling as stochastic simulation, with Monte Carlo being reserved for Monte Carlo integration and Monte Carlo statistical tests. Sawilowsky distinguishes between a simulation, a Monte Carlo method, and a Monte Carlo simulation: a simulation is a fictitious representation of reality, a Monte Carlo method is a technique that can be used to solve a mathematical or statistical problem, and a Monte Carlo simulation uses repeated sampling to obtain the statistical properties of some phenomenon (or behavior).

**History**

Markov Chain Monte Carlo method was invented in the late 1940s by Stanislaw Ulam, while he was working on nuclear weapons projects at the Los Alamos National Laboratory. The name *Monte Carlo* refers to the Monte Carlo Casino in Monaco where Ulam's uncle would borrow money from relatives to gamble. Monte Carlo methods were central to the simulations required for the Manhattan Project, though severely limited by the computational tools at the time. In the 1950s they were used at Los Alamos for early work relating to the development of the hydrogen bomb, and became popularized in the fields of physics, physical chemistry, and operations research. The theory of more sophisticated mean field type particle Monte Carlo methods had certainly started by the mid-1960s, with the work of Henry P. McKean Jr. on Markov interpretations of a class of nonlinear parabolic partial differential equations arising in fluid mechanics. An earlier pioneering article by Theodore E. Harris and Herman Kahn, was published in 1951, using mean field genetic-type Monte Carlo methods for estimating particle transmission energies. The origins of these mean field computational techniques can be traced to 1950 and 1954 with the work of Alan Turing on genetic type mutation-selection learning machines. The first heuristic-like and genetic type particle algorithm or estimating ground state energies of quantum systems is due to Jack H. Hetherington in 1984. It was in 1993, that Gordon et al., published in their seminal work the first application of a Monte Carlo resampling algorithm in Bayesian statistical inference. Further developments in this field were developed in 2000 by P. Del Moral, A. Guionnet and L. Miclo.

**Working**

The main idea behind this method is that the results are computed based on repeated random sampling and statistical analysis. The Monte Carlo simulation is, in fact, random experimentations, in the case that, the results of these experiments are not well known. Monte Carlo simulations are typically characterized by many unknown parameters, many of which are difficult to obtain experimentally. Monte Carlo simulation methods do not always require truly random numbers to be useful. Many of the most useful techniques use deterministic, pseudorandom sequences, making it easy to test and re-run simulations. The only quality usually necessary to make good simulations is for the pseudo-random sequence to appear "random enough" in a certain sense.  
What this means depends on the application, but typically they should pass a series of statistical tests. Testing that the numbers are uniformly distributed or follow another desired distribution when a large enough number of elements of the sequence are considered is one of the simplest and most common ones. Weak correlations between successive samples are also often desirable/necessary.  
All the input variables used for Monte Carlo simulations may not be correlated with each other. This will give inaccurate or less efficient results. In addition to that, Monte Carlo method takes up a lot of time and space. To remedy this, the data obtained about the different input variables are processed using Cholesky decomposition to correlate them and also decrease the computation time. When required to generate many correlated normal random variables, Cholesky decomposition allows you to simulate uncorrelated normal variables and transform them into correlated normal variables. This allows to save time dealing with a large multi-variate normal.

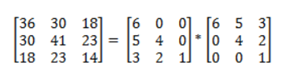
**Cholesky Decomposition**

Cholesky decomposition reduces a symmetric positive definite matrix into a lower-triangular matrix which when multiplied by its transpose produces the original symmetric matrix. It reduces a matrix in the form

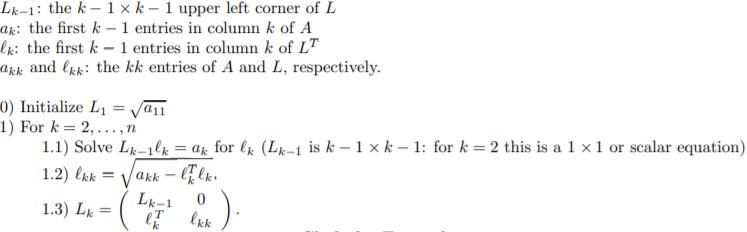
A= L LT

where L is a lower triangular matrix with real and positive diagonal entries, and LT denotes the conjugate transpose of L.

For example:



Cholesky decomposition is an iterative process. The algorithm for the decomposition is as follows:



Cholesky decomposition can solve systems of equations of any size in 2 steps.

Cholesky decomposition works only on symmetric positive definite matrix. If the matrix used is not symmetric positive definite, then either the diagonal element becomes 0 or complex.

**Positive Definite Matrix**

Positive definite matrix is a symmetric matrix with all positive eigenvalues and as we are talking about symmetric matrices the all the eigenvalues real value.

A *n x n* symmetric real matrix M is said to be positive definite if xT Mx ≥ 0 for all x in Rn. Formally,



**RESULTS AND DISCUSSION**

Monte Carlo simulation is a very useful mathematical technique for analyzing uncertain scenarios and providing probabilistic analysis of different situations. The basic principle for applying Monte Carlo analysis is simple and easy to grasp. In Monte Carlo simulation, we identify a statistical distribution which we can use as the source for each of the input parameters. Then, we draw random samples from each distribution, which then represent the values of the input variables. Markov Chain Monte Carlo method was invented in the late 1940s by Stanislaw Ulam, while he was working on nuclear weapons projects at the Los Alamos National Laboratory. To achieve the simulation by Monte Carlo method, we use Cholesky decomposition to process the matrices. A Cholesky matrix transforms a vector of uncorrelated (i.e. independent) normally-distributed random variates into a vector of correlated (i.e. dependent) normally-distributed random variates. These now correlated random variates can be used in a Monte Carlo simulation where correlated random variates are required. Cholesky decomposition reduces a symmetric positive definite matrix into a lower-triangular matrix which when multiplied by its transpose produces the original symmetric matrix. Positive definite matrix is a symmetric matrix with all positive eigenvalues and as we are talking about symmetric matrices the all the eigenvalues real value.

**SUMMARY AND CONCLUSION**

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